PAPER Special Section on Measurement Technologies for Microwave Materials, Devices and Circuits

Cut-Off Circular Waveguide Method for Dielectric Substrate Measurements in Millimeter Wave Range

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SUMMARY A novel resonator structure for the cut-off circular waveguide method is proposed to suppress the unwanted TE modes in the axial direction and TM modes in the radial direction. In this method, a dielectric plate sample is placed between two copper circular cylinders and clamped by two clips. The cylinder regions constitute the TE_{0m} mode cut-off waveguides. The measurement principle is based on a rigorous analysis by the Ritz-Galerkin method. Many resonance modes observed in the measurement can be identified effectively by mode charts. In order to verify the validity of the novel structure for this method, the temperature dependences for three low-loss organic material plates were measured in the frequency range 40 to 50 GHz. It is found that modified polyolefin plates have comparable electric characteristics and low price, compared with PTFE plates. Moreover, it is verified that the novel resonator structure is effective in improvement of accuracy and stability in measurement. The measurement precisions are estimated within 1 percent for ε_r and within 15 percent for $\tan\delta$.

key words: *cut-off circular waveguide method, millimeter wave, dielectric substrate measurement*

1. Introduction

Recently, the development of new material with low-loss characteristics and low price is requested for application to millimeter wave circuit. It has been an important subject to measure complex permittivity of dielectric materials accurately and efficiently in millimeter wave range. Some measurement methods [1]–[3] have been reported to evaluate these dielectric materials in millimeter wave range.

In our laboratory, we have proposed a cut-off circular waveguide method to measure the temperature dependence of complex permittivity of low-loss dielectric plates accurately and efficiently in the millimeter wave range [4]–[10].

At first, a TE_{011} mode circular waveguide method was proposed by S.B. Cohn and K.C. Kelly [11], where a resonator is constituted by inserting a circular disk sample into a TE_{01} mode cut-off circular waveguide. This method was applied with a waveguide excitation to the millimeter wave measurement [12]. In order to measure any size of samples nondestructively, a novel resonator structure, where a dielectric plate sample is placed between two copper circular cylinders, was proposed by Y. Kobayashi and J. Sato [4], and G. Kent [13], independently. However, it was found that the correction value of the fringe effect for relative permittivity by the G. Kent's method was not correct.

The features of the cut-off circular waveguide method are as follows,

- The measurement principle is based on rigorous analysis by the Ritz-Galerkin method with the mode matching technique [4], [5].
- The dielectric plate sample is placed between two cylinders into which a copper circular cylinder is cut in the middle of height and clamped by two clips; hence, the sample can be exchanged easily [6].
- The millimeter wave vector network analyzer constituted by a coaxial cable system is used; hence, it is easy to adjust the coupling strength finely.
- A mode chart is presented to identify many resonance modes observed in the measurement [7].
- An automatic measurement system was developed to measure the temperature dependence more efficiently and precisely [8].
- A grooved circular cavity for separating degenerate TE and TM modes to measure the dimension and relative conductivity accurately is presented [9].

Recently, we found that the unwanted TE modes in the axial direction and TM modes in the radial direction are excited in the conventional structure [14]. These resonance modes affect the complex permittivity measurements.

In this paper, a novel resonator structure for the cut-off circular waveguide method is proposed to suppress the unwanted TE modes in the axial direction and TM modes in the radial direction, respectively. The validity of correction for the fringe effect by the rigorous analysis is confirmed experimentally. In order to verify the validity of the novel resonator structure for this method, the temperature dependences of complex permittivity for three low-loss organic material plates are measured for the TE_{011} mode in the frequency range 40 to 50 GHz.

2. Measurement Principle

2.1 Resonator Structure

A novel resonator structure is shown in Fig. 1(a). A copper circular cylinder with the diameter D is cut into two parts in the middle of the height H. A dielectric plate sample having the thickness t and the diameter d, which is larger than D, is placed between these cylinders and clamped by two clips. The cylinder regions constitute the TE_{0m} mode cut-off waveguides; hence, the fields decay exponentially in the axial direction. Similarly, the dielectric plate region outside D

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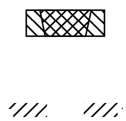


because the accurate value of D cannot measured mechanically. In this case, copper plates are attached at both ends in place of the conductor circular horns with wave absorbers. The degenerate TM_{11p} mode can be separated from the TE_{01p} mode by grooves machined at both ends of the cylinders [9]. The values of D and H are determined from a couple of resonance frequencies f_{0p} and f_{0q} measured for the TE_{01p} and TE_{01q} ($p \neq q$, $p < q$, integer) modes by using following equations [5],

$$D = \frac{c j'_{01}}{\pi} \sqrt{\frac{q^2 - p^2}{\left(q f_{0p}\right)^2 - \left(p f_{0q}\right)^2}} \tag{9}$$

$$H = \frac{c}{2} \sqrt{\frac{q^2 - p^2}{f_{0q}^2 - f_{0p}^2}} \tag{10}$$

where c is velocity of light and j'_{01}=3.83171.

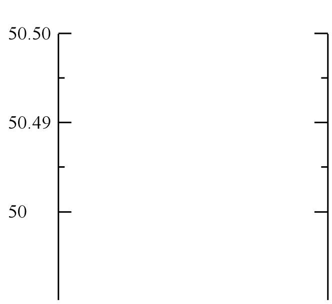
The value of σ_r is determined from the f_{0p} and Q_{up} values measured for the TE_{01p} mode by using following equation [5],

$$\sigma_r = \frac{4\pi f_{0p} Q_{up}^2 \left\{ j'^2_{01} + 2\,(p\pi)^2 \left(\frac{D}{2H}\right)^3 \right\}^2}{\sigma_0 \mu_0 c^2 \left\{ j'^2_{01} + \left(\frac{p\pi D}{2H}\right)^2 \right\}^3} \tag{11}$$

3. Discussions of the Fringe Effect

3.1 Convergence on the Solution

As the number of the matrix N and the variable K for Eq. (1) are increased, the solution approaches true values. In actual calculations, N and K are chosen so that the solution has an accuracy of five significant figures because of the reduction of calculation time. Figure 3 shows the calculation results of f_0 for ε_r=2 and t=0.5 mm with the TE_{0m1} ($m = 1, 2, 3$) modes. As the order of m is increased, the convergence is slower. Those behaviors of the convergence shows similar tendency. In case of N=20 and K=40 for TE_{011} mode, we obtain the accuracy of five significant figures for the solution of f_0. Moreover, calculation time is about 1 second using



stant within the error. On the other hand, ε_k is decreased, because the correction value is too much. In other word, Eq. (13) is not proper and the relational expression is expressed as follows,

$$\frac{\Delta\varepsilon_r}{\varepsilon_r} \cong \frac{1}{2}\frac{\Delta\varepsilon_k}{\varepsilon_r} \tag{13}$$

4. Discussion for the Resonator Structure

We discuss the resonator structure to suppress unwanted resonance modes. The unwanted resonance modes are excited in the axial and radial directions, respectively. The resonant frequencies of these modes are determined from relative permittivity and thickness of a dielectric plate.

4.1 Axial Direction

Two sapphire plates named as sample-A with t=0.298 mm and sample-B with t=0.506 mm are measured by using three resonators with the conventional absorbers [6]. The resonator I has D=6.991 mm, H=30.917 mm and σ_r=84.8%, the resonator II has D=6.985 mm, H=26.117 mm and σ_r=76.2% and the resonator III has D=6.480 mm, H=24.289 mm and σ_r=75.0%.

The measured results are shown in Fig. 5. It is found that the tan δ value of the sapphire-A measured using the resonator I is considerably higher than other.

In the two cases of conventional wave absorbers and copper plates attached both ends of the cylinder, the frequency responses for the resonator I with the sapphire-A plate are shown in Fig. 6 by the long dash line and the dash line, respectively. When the conventional wave absorbers are attached, other resonance modes are not observed. When the wave absorbers are exchanged to the copper plates, it is found that the unwanted cavity mode exists around 38.2 GHz. We call this mode the TE mode in the axial direction [14]. As a result, the measured Q_u value decreases due to the influence of the unwanted TE mode because these conventional wave absorbers do not have sufficient attenuation to suppress them completely.

The novel structure of the wave absorber parts in the resonator to suppress the unwanted TE modes is shown in Fig. 2(a). The sapphire plates are measured by using the resonator I with the novel wave absorber horns. The measured results are shown in Fig. 5 by the circular mark. The frequency response is shown in Fig. 6 by the solid line. It is found that the value of tan δ of sapphire-A measured by using the resonator I with the novel wave absorber horns agree with the other results within measurement error 15 percent. The novel wave absorber horns are useful to suppress the unwanted TE modes.

4.2 Radial Direction

The frequency responses of a modified polyolefin plate with t=1.165 mm by using the conventional [6] and novel type resonators are shown in Fig. 7 by dash line and solid line, respectively. The resonance frequencies calculated from the mode chart for a simple resonator neglecting the fringe effect [6] are indicated on the top of Fig. 7. It is found the small resonance peaks at 12, 16, 30, 38, 41 and 48 GHz, which are not calculated from the mode chart, are excited by using the conventional resonator. We call these modes the TM modes in the radial direction [14]. These modes affect to measured value of the unloaded Q when these modes is close to the TE_{011} mode. On the other hand, the small resonance peaks are suppressed by using the novel resonator and the measured resonance frequencies agree with the one calculated from the mode chart. The novel resonator is useful to suppress the unwanted TM modes.

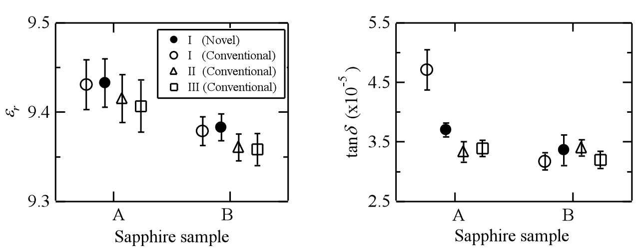


Fig. 5 Measured results for two sapphire plates (sample-A: t=0.298 mm and sample-B: t=0.506 mm).

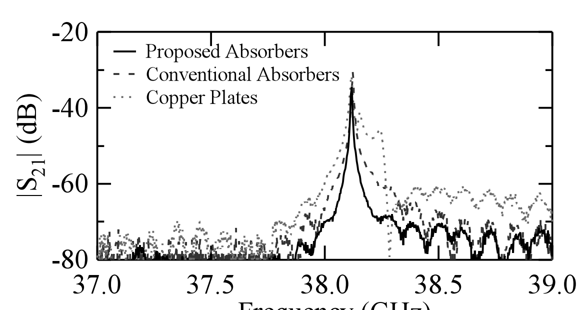
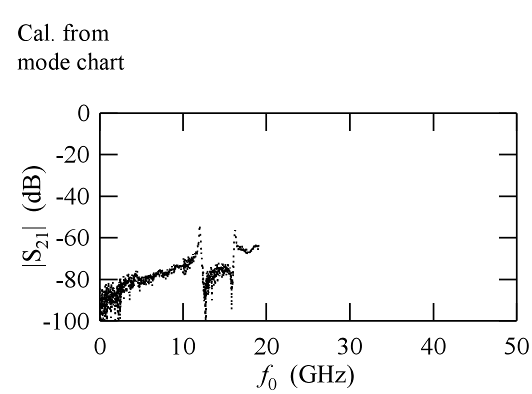


Fig. 6 The frequency responses for the resonator I with the sapphire-A plate attaching two type wave absorbers or copper plates.



5. Identification of Resonance Modes

A program to make a mode chart for f_0 versus ε_a was developed on the basis of the characteristic equations [6] for a simple cut-off circular waveguide resonator without the fringe effect, as shown in Fig. 8, where f_0 is the measured resonance frequency and ε_a is an approximate relative permittivity when the fringe effect is neglected. The mode chart for D=6.991 mm and t=2.050 mm for a modified polyolefin

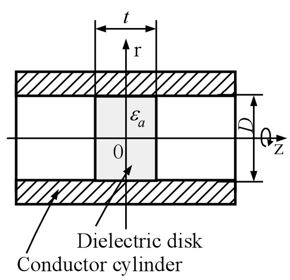


Fig. 8 The cross sectional view of a simple circular cut-off waveguide resonator where the fringe effect is neglected.

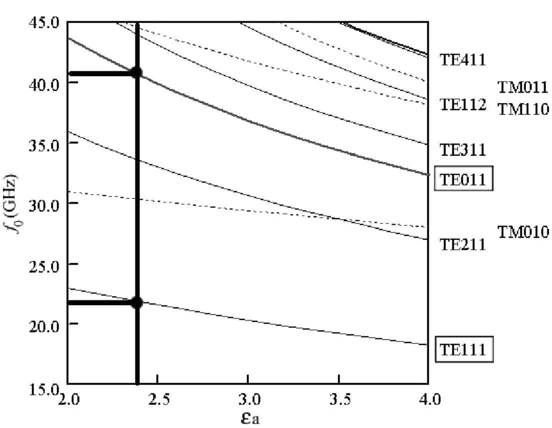


Fig. 9 Mode chart of a dielectric disk resonator loaded in a circular cut-off waveguide calculated for D=6.991 mm and t=2.050 mm for a modified polyolefin plate.

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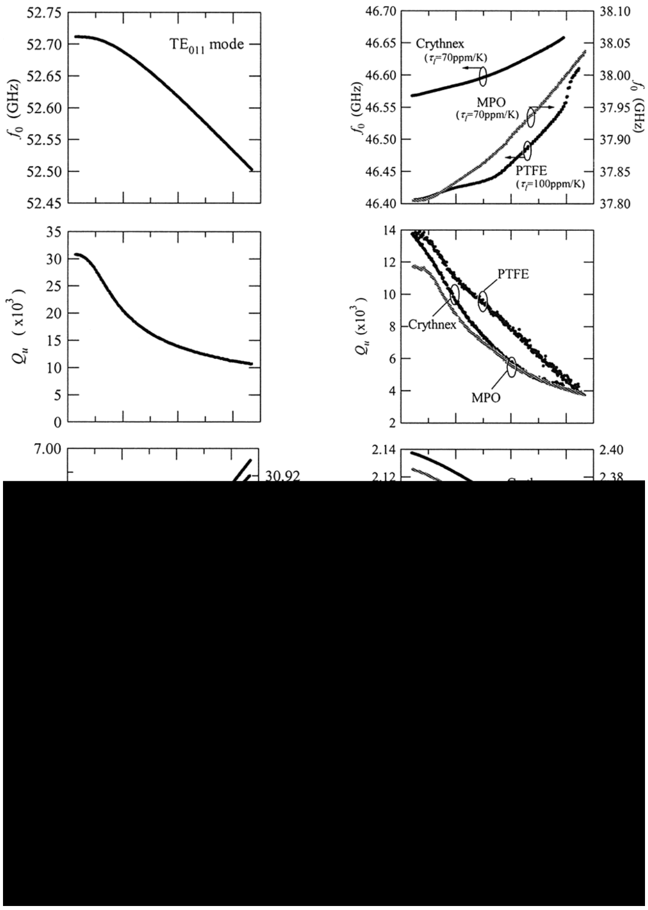

[redacted]

(a) Circular empty cavity. (b) Organic plates.

Fig. 12 Temperature dependences of circular empty cavity and PTFE, Crythnex, and MPO plates.

measured for the TE_{011} mode. The measured results are shown in Fig. 12(b).

The f_0 and ε_r of the PTFE plate have inflection points near 50 K, 170 K and 290 K, because of the phase transitions of crystal construction. However, the f_0 and ε_r of the MPO plate have no inflection point. Moreover, the $\tan\delta$ value of the MPO plate is quite lower than that of the PTFE above room temperature. We can expect that the MPO plates have the high possibility for application to millimeter wave circuit, because of comparable electric characteristics and low price, compared with PTFE plates.

7. Conclusion

It was verified that the novel resonator structure proposed in this paper was effective in improvement of accuracy and stability in measurement. As a result, it is concluded that the cut-off circular waveguide method is useful to measure the temperature dependence of complex permittivity of low-loss dielectric plates accurately and efficiently in millimeter wave range. The measurement precisions are estimated within 1 percent for $\varepsilon_r = 2$–30 and within 15 percent for $\tan\delta = 10^{-3}$–10^{-6}.

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Appendix

A.1 Analysis of Resonance Frequency f_0

Figure A· 1 shows a resonator structure used in this rigorous analysis. The circular cylinder is cut into two parts in the middle of the height H. A dielectric plate having relative permittivity ε_r, thickness t and diameter d, which is a larger size than the diameter of the cylinder D, is sandwiched between two dielectric supports having relative permittivity ε_g and thickness g. They are placed the two cup-shaped circular cylinders. This structure corresponds to Fig. A· 1 when g=0. The relative permeability μ_r=1 is assumed in each medium. When $g \neq 0$ in this configuration, we can calculate the conductor loss at the cavity fringe by the perturbation of g [15], [16].

The TE_{0m1} resonance mode can be analyzed rigorously by the Ritz-Galerkin method. From the structural symmetry, it is sufficient to consider only the region $0 \le z \le H_1$. The region is divided into three homogeneous subregions [I], [II], and [III]. The quantities for the subregions are denoted by subscripts 1, 2, and 3, respectively. A time harmonic factor $e^{j\omega t}$ is omitted. Applying the boundary conditions on the r-θ plane at $z = 0$ and on the conducting surface. We can expand magnetic Helmholtz vector Π_m for each region as follows:

$$\begin{aligned}
\Pi_{m1} &= \sum_{p=1}^{\infty} A_p J_0\left(k_{r1p} r\right)\cos\beta_{1p} z \\
\Pi_{m2} &= \sum_{p=1}^{\infty} B_p J_0\left(k_{r1p} r\right)\cos\beta_{2p} z \\
&\quad + \sum_{p=1}^{\infty} C_p J_0\left(k_{r1p} r\right)\sin\beta_{2p} z \\
\Pi_{m3} &= \sum_{q=1}^{\infty} D_q J_0\left(k_{r3q} r\right)\sin\beta_{3q}\left(H_1 - z\right)
\end{aligned} \tag{A· 1}$$

where

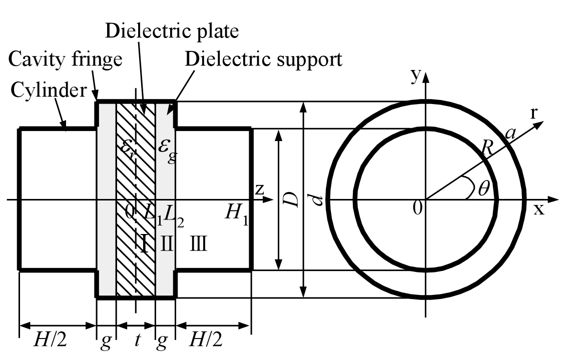

Fig. A· 1 Geometry of analysis.

$$\begin{aligned}
\beta_{1p}^2 &= \varepsilon_r k_0^2 - k_{r1p}^2 \\
\beta_{2p}^2 &= \varepsilon_g k_0^2 - k_{r1p}^2 \\
\beta_{3q}^2 &= k_0^2 - k_{r3q}^2
\end{aligned} \tag{A· 2}$$

$$\begin{aligned}
k_{r1p} &= u_p / a \\
k_{r3q} &= v_q / R
\end{aligned} \tag{A· 3}$$

In the above, A_p, B_p, C_p, and D_q are expansion coefficients to be determined from the boundary conditions for the regions [I], [II], and [III]. Moreover, the electromagnetic fields components of the TE_{0m1} mode in each region are obtained by substituting (A· 1) into (A· 4).

$$\begin{aligned}
H_z &= k^2 \Pi_m + \frac{\partial^2 \Pi_m}{\partial z^2} \\
H_r &= \frac{\partial^2 \Pi_m}{\partial r \partial z} \\
E_\theta &= j\omega\mu_0 \frac{\partial \Pi_m}{\partial r}
\end{aligned} \tag{A· 4}$$

The relationship of B_p and C_p is determined from the continuity of H_r at $z = L_1$ as follows:

$$\frac{C_p}{B_p} = -\frac{\beta_{1p}\tan\beta_{1p}L_1 - \beta_{2p}\tan\beta_{2p}L_1}{\beta_{1p}\tan\beta_{1p}L_1\tan\beta_{2p}L_1 + \beta_{2p}} \tag{A· 5}$$

The relationship of A_p and D_q is determined from the continuity of E_θ and H_r at $z = L_2$. From the former case, we first obtain

$$\begin{aligned}
&\sum_{p=1}^{\infty} -j\omega\mu_0 A_p k_{r1p} J_1\left(k_{r1p} r\right) \\
&\quad \cdot \left(B_p \cos\beta_{2p}L_2 + C_p \sin\beta_{2p}L_2\right) = E_\theta(r)
\end{aligned} \tag{A· 6}$$

$$\sum_{q=1}^{\infty} -j\omega\mu_0 D_q k_{r3q} J_1\left(k_{r3q} r\right)\sin\beta_{3q}(H/2) = E_\theta(r) \tag{A· 7}$$

where $E_\theta(r)$ is the r component of unknown electric field at $z = L_2$. Multiplying $rJ_1\left(k_{r1p} r\right)$ on both sides of (A· 6) and integrating from 0 to a with respect r. Also, multiplying $rJ_1\left(k_{r3q} r\right)$ on both sides of (A· 7) and integrating from 0 to R with respect r. We obtain the following expressions from the orthogonality of Bessel functions and $E_\theta(r)$=0 from R to a:

$$\begin{aligned}
&-j\omega\mu_0 A_p k_{r1p} \frac{a^2}{2} \left\{ \begin{array}{c} J_0^2\left(u_p\right) \\ J_1^2\left(u_p\right) \end{array} \right\} \\
&\quad \cdot \left(B_p \cos\beta_{2p}L_2 + C_p \sin\beta_{2p}L_2\right) \\
&\quad = \int_0^R E_\theta(r)\, r J_1\left(k_{r1p} r\right) dr
\end{aligned} \tag{A· 8}$$

$$\begin{aligned}
&-j\omega\mu_0 D_q k_{r3q} \sin\beta_{3q}(H/2)\, \frac{R^2}{2} J_0^2\left(v_q\right) \\
&\quad = \int_0^a E_\theta(r)\, r J_1\left(k_{r3q} r\right) dr
\end{aligned} \tag{A· 9}$$

From the latter case, we then obtain

$$\sum_{p=1}^{\infty} A_p k_{r1p} \beta_{2p} J_1\left(k_{r1p} r\right) \cdot \left(B_p \sin\beta_{2p} L_2 - C_p \cos\beta_{2p} L_2\right) = \sum_{q=1}^{\infty} D_q k_{r3q} \beta_{3q} J_1\left(k_{r3q} r\right) \cos\beta_{3q}(H/2) \tag{A· 10}$$

Substituting (A· 8) and (A· 9) into (A· 10) to eliminate A_p and D_q, multiplying $rJ_1\left(k_{r3q}r\right)$ on both sides and integrating from 0 to R with respect of r, we obtain the following integral equation for $E_\theta(r)$.

$$\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \beta_{2p} \frac{\tan\beta_{2p}L_2 - \dfrac{C_p}{B_p}}{1 + \dfrac{C_p}{B_p}\tan\beta_{2p}L_2} \frac{P_{pq}}{a^2 \left\{ \begin{array}{c} J_0^2\left(u_p\right) \\ J_1^2\left(u_p\right) \end{array} \right\}} \cdot \int_0^R E_\theta(r)\, r J_1\left(k_{r1p} r\right) dr = \sum_{q=1}^{\infty} \frac{\beta_{3q}}{2} \cot\beta_{3q}(H/2) \int_0^a E_\theta(r)\, r J_1\left(k_{r3q} r\right) dr \tag{A· 11}$$

We apply the Ritz-Galerkin method to solve (A· 11) by numerical analysis. And expanding $E_\theta(r)$ into an eigenfunction of region [III] as follows,

$$E_\theta(r) = \sum_{l=1}^{\infty} E_l J_1\left(k_{r3l} r\right), \quad k_{r3l} = \frac{v_l}{R} \tag{A· 12}$$

where E_l is an expansion coefficient. Substituting (A· 12) into (A· 11). Moreover, approximating infinite sum to finite sum, we obtain the following homogeneous equation for E_l with $l, q = 1, 2 \cdots N$, $p = 1, 2 \cdots K$.

$$\sum_{l=1}^{N}\sum_{q=1}^{N} \left[\frac{\delta_{lq}}{4} \beta_{3q} R^2 J_0^2\left(v_q\right) \cot\beta_{3q}(H/2) - \sum_{p=1}^{K} \frac{\beta_{2p} P_{pq} P_{pl}}{a^2 \left\{ \begin{array}{c} J_0^2\left(u_p\right) \\ J_1^2\left(u_p\right) \end{array} \right\}} \frac{\tan\beta_{2p}L_2 - \dfrac{C_p}{B_p}}{1 + \dfrac{C_p}{B_p}\tan\beta_{2p}L_2} \right] E_l = 0 \tag{A· 13}$$

For E_l, which is not zero in (A· 13), the determinant of the coefficient matrix needs to be zero. Accordingly, this requirement yields the following $N \times N$ square determinant with variable K as a characteristic equation for the TE_{0m1} mode.

$$\det H_{lq}\left(f_0; \varepsilon_r, \varepsilon_g, g, t, d, D, H\right) = 0 \tag{A· 14}$$

where the matrix elements H_{lq} are following

$$H_{lq} = \frac{\delta_{lq}}{4} Y_q J_0^2\left(v_q\right) \cot\left(Y_q \frac{(H/2)}{R}\right) - \sum_{p=1}^{K} \frac{Z_p P_{pq} P_{pl}}{\dfrac{a^2}{R^2} \left\{ \begin{array}{c} J_0^2\left(u_p\right) \\ J_1^2\left(u_p\right) \end{array} \right\}} \frac{\tan\left(Z_p \dfrac{L_2}{R}\right) - \dfrac{C_p}{B_p}}{1 + \dfrac{C_p}{B_p}\tan\left(Z_p \dfrac{L_2}{R}\right)} \tag{A· 15}$$

$$X_p^2 = \varepsilon_r (k_0 R)^2 - \left(\frac{R}{a} u_p\right)^2$$

$$Z_p^2 = \varepsilon_g (k_0 R)^2 - \left(\frac{R}{a} u_p\right)^2 \tag{A· 16}$$

$$Y_q^2 = (k_0 R)^2 - v_q^2$$

$$P_{pq} = \frac{v_q J_0\left(v_q\right) J_1\left(\dfrac{R}{a} u_p\right)}{\left(\dfrac{R}{a} u_p\right)^2 - v_q^2} \tag{A· 17}$$

$$P_{pl} = \frac{v_l J_0\left(v_l\right) J_1\left(\dfrac{R}{a} u_p\right)}{\left(\dfrac{R}{a} u_p\right)^2 - v_l^2} \tag{A· 18}$$

Here, in case of subregions [I] and [II] are propagate region and subregions [III] is cut-off region, we obtain following equation,

$$H_{lq} = \frac{\delta_{lq}}{4} Y_q' J_0^2\left(v_q\right) \coth\left(Y_q' \frac{(H/2)}{R}\right) - \sum_{p=1}^{K} \frac{Z_p P_{pq} P_{pl}}{\dfrac{a^2}{R^2} \left\{ \begin{array}{c} J_0^2\left(u_p\right) \\ J_1^2\left(u_p\right) \end{array} \right\}} \frac{\tan\left(Z_p \dfrac{L_2}{R}\right) - \dfrac{C_p}{B_p}}{1 + \dfrac{C_p}{B_p}\tan\left(Z_p \dfrac{L_2}{R}\right)} \tag{A· 19}$$

where

$$Y_q'^2 = v_q^2 - (k_0 R)^2 \tag{A· 20}$$

Furthermore, we consider height H is infinite in Eq. (A· 19). We obtain following equation,

$$H_{lq} =$$

$$\frac{1}{Q_c} = \frac{1}{Q_{cy}} + \frac{1}{Q_{cg}} \tag{A·23}$$

where Q_{cy}, and Q_{cg} are ones due to the conductor losses of cylinder and cavity fringe, respectively. They are expressed by

$$Q_{cy} = \frac{f_0}{(-\Delta f_{0D}/\Delta D)\,\delta_c} \tag{A·24}$$

$$Q_{cg} = \frac{f_0}{\left(-\Delta f_{0g}/\Delta g\right)\delta_c} \tag{A·25}$$

$$\delta_c = \frac{1}{\sqrt{\pi f_0 \mu_0 \sigma}} \tag{A·26}$$

where δ_c is a skin depth, $\sigma = \sigma_0\sigma_r$ is the conductivity, $\sigma_0 = 58\times10^6$ S/m is the conductivity of the standard copper, and $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability in the vacuum. Also, Q_d is given by

$$Q_d = \frac{1}{\tan\delta}\frac{f_0}{2\varepsilon_r\cdot(-\Delta f_{0\varepsilon}/\Delta\varepsilon_r)} \tag{A·27}$$

where the resonance frequency change Δf_{0x} due to a small distance change Δx can be calculated from (A· 14), where x is D, H, g or ε_r. As a result, Eq. (2) is derived from (A· 22) to (A· 27).

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