# 50GHz measurements of temperature dependence of complex permittivity of dielectric plates by a cut-off circular waveguide method

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*Abstract* — A novel cut-off circular waveguide method with coaxial excitation is proposed to measure the temperature dependence of complex permittivity of low loss dielectric plates in the millimeter wave range. Measurement principal is based on a rigorous analysis by the mode matching method. The automatic measurement system applicable to 60GHz is constructed using 2.2mm semi-rigid cable and V connectors. The temperature dependencies for PTFE and Crythnex plates were measured in the temperature range 20 to 300K. It is verified that this method is useful for a precise measurement of the complex permittivity of dielectric plates.

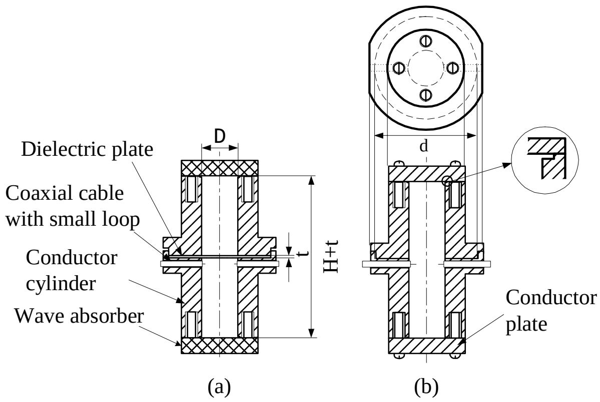
## I. Introduction

As conventional techniques of measuring low loss dielectric materials in millimeter wave region, an open resonator method and a cavity resonance method with waveguide excitation are known [1][2]. However, these methods are not suitable to measure the temperature dependences, because a measurement apparatus is mechanically unstable for temperature change in a cryostat or an oven for the open resonator method, and the adjustment of coupling strength by a waveguide is not practical for the cavity resonance method. Presently, we can use millimeter wave vector network analyzers constituted by a coaxial cable system. The excitation of a cavity by a coaxial cable makes this measurement easy over the wide frequency band and to adjust the coupling strength finely.

In this paper, a novel cut-off circular waveguide method based on a rigorous analysis by the mode matching technique is proposed to measure the temperature dependences of complex permittivity of low loss dielectric plates at 50GHz. The design of a cavity to measure the complex permittivity is also discussed. By this method, the temperature dependences of complex permittivity for PTFE and Crythnex plates will be measured at 45GHz.

## II. Measurement principal

A resonator structure used in this measurement is shown in Fig. 1. A circular cylinder resonator clamping a dielectric plate is shown in Fig. 1(a). This cylinder is cut into two parts in the middle of the height *H*. A dielectric plate sample having the thickness *t* and a larger size than the diameter *D* is placed between these cylinders and clamped by two clips; hence a sample to be measured can be quickly removed and replaced by another one. The cylinders constitute the $TE_{0m}$ mode (m: integer) cutoff waveguides; hence the fields decay exponentially on each side of the sample. The wave absorbers attached at both ends are needed to eliminate the unwanted higher order modes.




$$\det F\left(f_0;\varepsilon_r,\varepsilon_g,t,g,D,H,d\right)=0 \quad \cdots\cdots\cdots\cdots \quad (1)$$

$$\tan\delta = A/Q_u - BR_s \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

where

$$A=-f_0/\left[2\varepsilon_r\cdot\left(\Delta f_0/\Delta\varepsilon_r\right)\right] \quad \cdots\cdots\cdots\cdots \quad (3)$$

$$B=\frac{1}{120\pi k_0\varepsilon_r}\frac{1}{\left(\Delta f_0/\Delta\varepsilon_r\right)}\left(\frac{\Delta f_0}{\Delta H}+\frac{\Delta f_0}{\Delta R}+\frac{\Delta f_0}{\Delta g}\right) \cdots (4)$$

$$R_s=\sqrt{\pi f_0\mu_0/\sigma} \quad (\Omega) \qquad \mu_0=4\pi\times10^{-7} \ (H/m)$$

$$\sigma=\sigma_0\sigma_r \qquad \sigma_0=58\times10^6 \ (S/m)$$

(c)

Fig. 1. Cross sectional view of a resonator structure.

(a) Circular cylinder resonator clamping a dielectric plate.
(b) Circular empty cavity.
(c) Measurement formulas.

The values of $D$, $H$ and $\sigma_r$ of a copper cavity are measured using the $TE_{01}$ mode of an empty cavity structure shown in Fig. 1(b), where copper plates are attached at both ends in place of wave absorbers. The $D$ and $H$ are calculated from two resonance frequencies $f_0$ for the $TE_{01p}$ and $TE_{01q}$ modes ($p \neq q$, integer), and the effective relative conductivity $\sigma_r$, including influence of oxidation and roughness of the copper surface, is determined from an unloaded $Q$, $Q_u$ measured for the $TE_{01p}$ mode [3]. The degenerate $TM_{11p}$ mode can be separated from the $TE_{01p}$ mode by grooves machined at each end of the cylinders.

These resonators are excited and detected by a pair of UT-47 semi-rigid coaxial cables (outer diameter 1.2mm) with a small loop at the top, which are set near the dielectric plate sample.

The value of relative permittivity $\varepsilon_r$ and loss tangent $\tan\delta$ of the sample, in consideration of the fringe effect, can be calculated accurately from the measured value of $f_0$ and $Q_u$ of the $TE_{0m1}$ mode, by using measurement formulas based on rigorous analysis by the mode matching technique, as shown in Fig. 1(c), where constants A and B are calculated from the frequency changes due to each perturbation by using eq. (1) [4].

## III. Design of Cavity

A mode chart of an empty cavity shown in Fig. 2 was calculated from

$$\left(\frac{f_0 D}{c}\right)^2 = \frac{1}{4}\left(\frac{pD}{H}\right)^2 + \left(\frac{j_{nm}^{(\prime)}}{\pi}\right)^2 \qquad (5)$$

for the $TE_{nmp}$ mode with $j'_{nm}$ and for the $TM_{nmp}$ mode with $j_{nm}$, where c is the velocity of light, and $j_{nm}$ and $j'_{nm}$ are mth zero values of the nth Bessel function of the first kind and its differentiation.

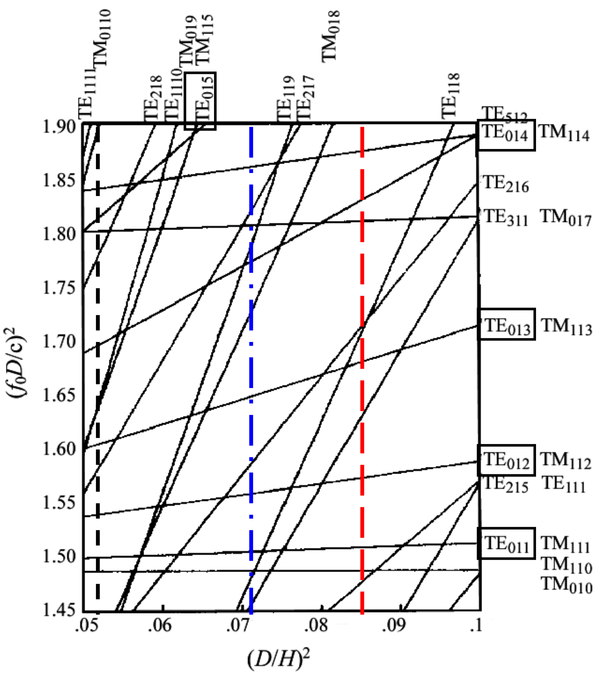


Fig. 2 The mode chart of an empty cavity resonator

At first, $D$ was determined to be 7.0mm so that the resonance frequency of the $TE_{011}$ mode of the empty cavity becomes just below 50GHz. Then the ratio $(D/H)^2$ were determined to be 0.0510, 0.0718 and 0.0864 so that unwanted modes do not appear near the $TE_{01p}$ modes as indicated by broken and dot-dash lines in Fig. 2. Thus the values of $H$ were determined to be 31.0mm, 26.1mm and 23.8mm, respectively

## IV. Measured Results

### A. The influence of short-circuited plates

The experiments for the mode identification were performed for two resonators with a PTFE plate attaching Cu plates and wave absorbers at both ends. The measured results are shown in Fig. 3(a) for the former case and in Fig. 3(b) for the later case. The resonance frequencies calculated from the mode chart are indicated on the top of Fig. 3.

In Fig. 3(a), it is difficult to identify the resonance modes, because the many higher modes were existed. On the other hand, in Fig. 3(b), it is easy to identify the resonance modes, because the resonance modes appear little. Thus, it was found that the wave absorbers attached at both ends are useful to eliminate the unwanted cavity modes.

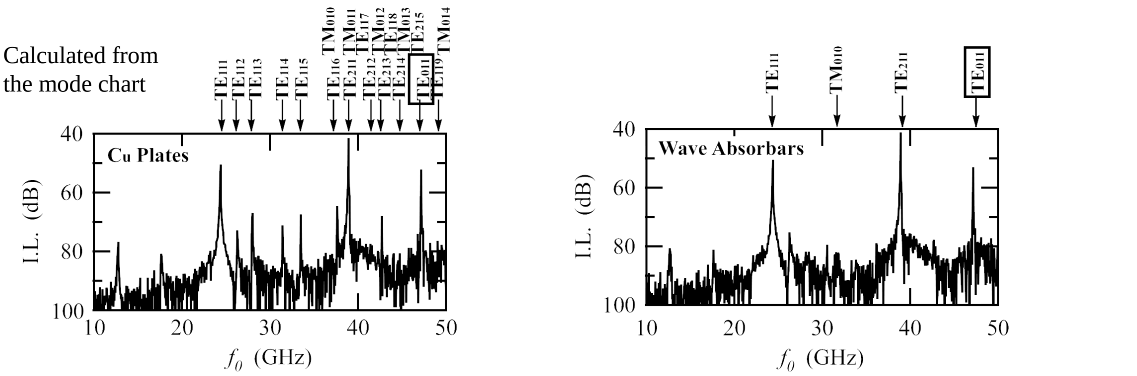
### B. Measured results using four cavities

Four cavities, which were numbered as 50KA01, 50KA02, 50KC01 and 50KC02, were manufactured to investigate the scatter of measured results of a dielectric plate sample.

Measured results for these four empty cavities are shown in Table 1. The values of $D$ and $H$ indicate the averages of ones calculated from some sets of $f_0$ measured for the $TE_{01p}$ and $TE_{01q}$ modes ($p \neq q$, integer). The value of $\sigma_r$ of the 50KA01 cavity was determined from the measured $Q_u$ value for the $TE_{013}$ mode and the $TE_{011}$ mode was used for the other three cavities.

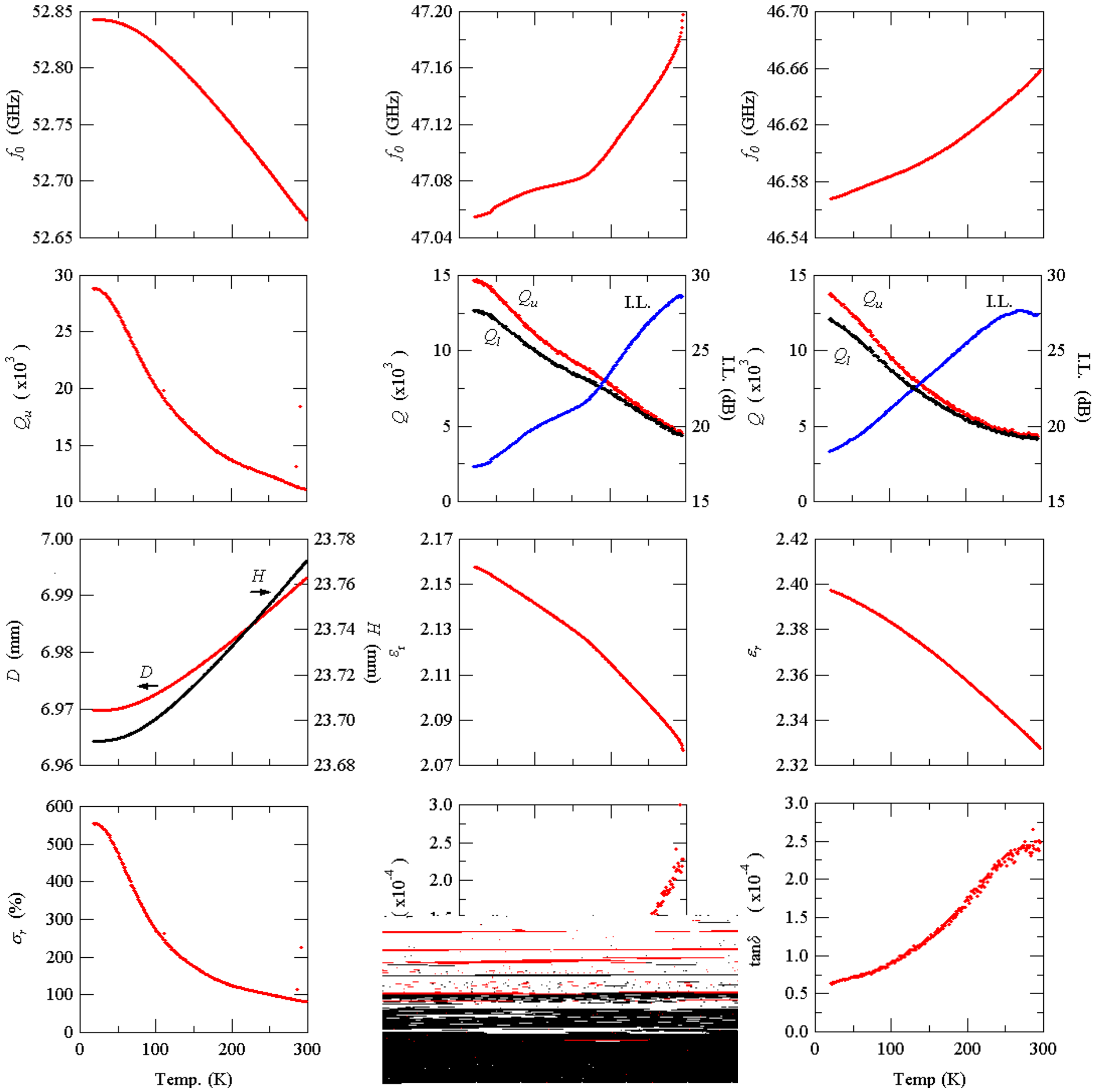
Table 1 Measured results for the empty cavities at 25℃

| Cavity | $f_0$ (GHz) | $Q_u$ | $D$ (mm) | $H$ (mm) | $\sigma_r$ (%) |
|---|---|---|---|---|---|
| 50KA01 ($TE_{013}$) | 54.300 ±0.002 | 10180 ±60 | 6.985 ±0.001 | 31.150 ±0.010 | 61.0 ±0.7 |
| 50KA02 | 52.649 ±0.001 | 11360 ±100 | 6.985 ±0.002 | 26.118 ±0.106 | 86.2 ±1.5 |
| 50KC01 | 52.520 ±0.001 | 11300 ±70 | 6.990 ±0.002 | 30.917 ±0.079 | 87.1 ±0.9 |
| 50KC02 | 52.647 ±0.001 | 11250 ±40 | 6.993 ±0.006 | 23.770 ±0.258 | 84.6 ±1.5 |




(a) Attaching Cu plates at both ends (b) Attaching wave absorbers at both ends

Fig. 3. The frequency response for the resonator with a PTFE plate ($t$=0.930mm) attaching Cu plates or wave absorbers at both ends.




(a) empty cavity (b) PTFE plate (c) Crythnex plate

Fig. 4. Temperature dependences of empty cavity (50KC02), PTFE plate and Crythnex plate.

The $\varepsilon_r$ and tanδ values of a Crythnex™ plate (Fujitsu Quantum Device Co., 10x10mm$^2$, $t$=0.823 ± 0.006mm and the linear thermal expansion coefficient $\tau_l$=70ppm/K) were measured by using the $TE_{011}$ mode of these four resonators. The results are shown in Table 2.

The value of $\varepsilon_r$ using the 50KA01 cavity is a little higher than ones using the other three cavities. We expect that this is due to the influence of gaps between the cylinder and the dielectric plate sample.

Table 2 Measured results for Crythnex plate ($t$=0.823mm, for $TE_{011}$, 25℃)

| Cavity | $f_0$ (GHz) | $Q_u$ | $\varepsilon_r$ | tan δ (x$10^{-4}$) |
|---|---|---|---|---|
| 50KA01 | 46.459 ±0.004 | 4070 ±170 | 2.375 ±0.007 | 2.40 ±0.18 |
| 50KA02 | 46.730 ±0.003 | 4130 ±80 | 2.322 ±0.010 | 2.74 ±0.08 |
| 50KC01 | 46.645 ±0.003 | 4240 ±60 | 2.332 ±0.010 | 2.62 ±0.06 |
| 50KC02 | 46.644 ±0.001 | 4110 ±50 | 2.330 ±0.012 | 2.74 ±0.05 |

### *C. Temperature dependence*

An automatic measurement system was developed to measure the temperature dependence in our laboratory. This system consists of Agilent technology network analyzer; 2.2mm semi-rigid coaxial cables with V connectors, Cryostat, and Windows personal computer with GP-IB. The automatic measurements of $\varepsilon_r$, tanδ and $\sigma_r$ are performed at each temperature change of 1K, using programs developed for HP-BASIC/WINDOWS.

The temperature dependences of $D$, $H$ and $\sigma_r$ for the 50KC02 cavity were measured using the $TE_{011}$ mode. The results are shown in Fig. 4(a).

Then the temperature dependences of $\varepsilon_r$ and tan$\delta$ of a PTFE plate (10x10mm$^2$, $t$=0.930mm and $\tau_l$=100ppm/K) and the Crythnex plate were measured using the $TE_{011}$ mode at 45GHz. The results are shown in Fig. 4(b) and (c). The $f_0$ of the PTFE plate have inflection points near 50K, 170K and 290K, because of phase transitions of crystal construction. The tanδ values of the Crythnex plate become approximately constant around the room temperature.

## V. CONCLUSIONS

It was verified that this method is useful to measure the temperature dependence of complex permittivity of low loss dielectric plates accurately and efficiently in millimeter wave region.

Crythnex plates have the high possibility to apply in millimeter wave circuit, because they have excellent electric characteristics comparable to PTFE plates and their price is much cheaper than one for PTFE plates.

## ACKNOWLEDGEMENT

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