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Design of a Grooved Circular Cavity for Separating Degenerate TE and TM Modes in Dielectric Substrate Measurements

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Abstract **A grooved circular cavity is designed for the measurements of dielectric substrates. The grooves are introduced to separate the degenerate TE_{01p} and TM_{11p} modes in circular cavities. A rigorous mode-matching method is used to investigate the influence of grooves on both the TE_{01p} and TM_{11p} modes. The dimensions of the grooves are determined based on the obtained numerical results. Comparative experiments of circular cavities with and without grooves validate the design method.**

***Key word*: dielectric substrate measurement, cut-off circular waveguide method, millimeter wave**

1 Introduction

Rapid progress of microwave and millimeter wave circuits requests cheap and low-loss dielectric materials. We have proposed the cavity resonator method [1, 2] and the cut-off circular waveguide method [3, 4] to measure the complex permittivities of low-loss dielectric substrates. In these methods, it is needed to separate the degenerate TM_{11p} mode from the TE_{01p} mode, because the dimensions and relative conductivity of the circular cavity can be measured from the TE_{01p} modes.

In this paper, a grooved circular cavity is designed for the measurements of dielectric substrates. The grooves are introduced to separate the degenerate TE_{01p} and TM_{11p} modes in circular cavities. A rigorous mode-matching method is used to investigate the influence of grooves on both the TE_{01p} and TM_{11p} modes. The dimensions of the grooves are determined from the results calculated numerically. The measured results validate the design method.

2 Analysis

A cross sectional view of a circular cavity with diameter D and height H is shown in Fig. 1. The circular cavity is cut into two parts in the middle of the height for clamping a dielectric plate sample. At both upper and lower ends of the cavity, grooves with depth d_c and width w are cut at both ends of the cylinder for separating the degenerate TE_{01p} and TM_{11p} modes. The cavity with these grooves can be viewed as coaxially cascaded circular waveguides with different diameters. Then electromagnetic (EM) fields in each of these circular waveguides are expressed by series of incident and reflected normal modes of the respective circular waveguide. At the step-junction between two neighbouring circular waveguides with different diameters, the EM boundary conditions are applied. As a result, the generalized scattering matrix of the incident and reflected normal modes including higher order modes is obtained at the step-junction. By combining the generalized scattering matrices at each of the step-junctions, and using the EM boundary conditions at the top and bottom of the cavity, we get finally the eigenvalue matrix equation for calculating the resonant frequencies and field distributions of different resonant modes [5].

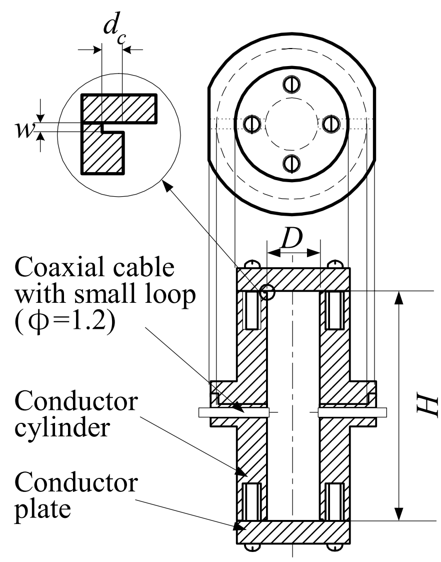


Fig. 1 Cross sectional view of a circular cavity

3 Design of the Cavity

3.1 Determination of the diameter and height of the circular cavity without grooves

For the circular cavity as shown in Fig. 1, but without grooves, the resonant frequencies f_0 are calculated by

$$\left(\frac{f_0 D}{c}\right)^2 = \frac{1}{4}\left(\frac{pD}{H}\right)^2 + \left(\frac{j_{nm}^{(')}}{\pi}\right)^2 \qquad (1)$$

where c is the velocity of light, and j_{nm} and j'_{nm} are the m-th root of the n-th Bessel function of the first kind and its

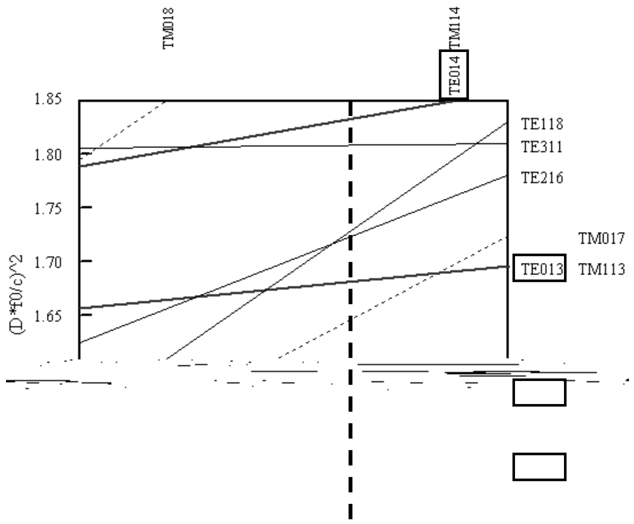


Fig. 2 The mode chart for a circular cavity

differential, respectively. j'_{nm} is used in (1) for the TE_{nmp} modes and j_{nm} for the TM_{nmp} modes.

At first, the ratio *D/H* were chosen as 0.294 from Fig. 2, so that unwanted modes do not appear near the TE_{01p} modes. Then the value of *D* is chosen as 7.0mm so that the resonant frequency of the TE_{011} mode becomes approximately 50GHz. Thus, the value of *H* is determined to be 23.8mm.

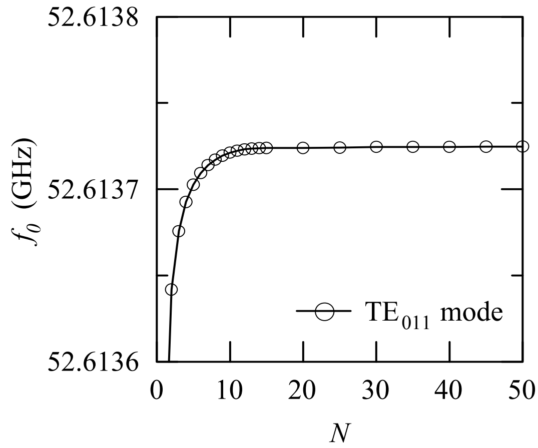
3.2 Determination of the size of grooves at both ends of the cylinder

We need to separate the degenerate TM_{11p} mode from the TE_{01p} mode, because the dimension and relative conductivity of a circular cavity are measured by using the TE_{01p} mode. This is realized by grooves machined at both ends of the cylinder as shown in Fig. 1. The groove with depth d_c and width w can be considered as the radial waveguides. The TE_{01} mode is cut off in the grooves; hence the resonant frequency of the TE_{01p} mode is affected little by the grooves. On the other hand, the TM_{11p} mode propagates forth and back in the grooves; hence the resonant frequency of the TM_{11p} mode is affected significantly.

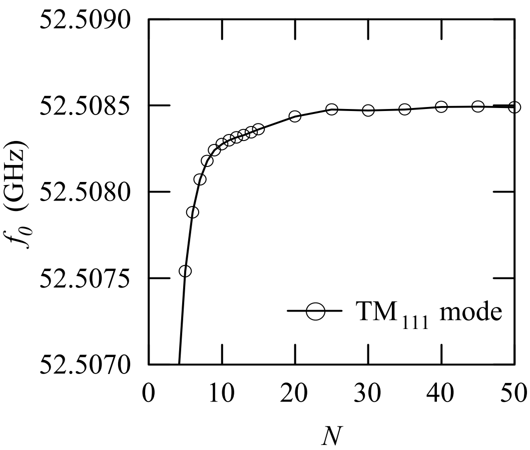
The resonant frequencies of the TE and TM modes are calculated by a computer program developed based on the mode-matching method described in Section 2.

At first, we check the convergence of the solution with number of expansion modes *N*. When D=7.0mm, H= 23.8mm, d_c=0.2mm and w=0.2mm, the calculated resonant frequencies for the TE_{011} and TM_{111} modes are shown in Fig. 3. It is seen that the solution converges to the sixth effective figure when $N \geq 25$.

Secondly, the f_0 values for the TE_{01p} and TM_{11p} modes were calculated as functions of d_c and w. The calculated results are shown in Fig. 4, where the values of f_0 changed by grooves are normalized by the values of f_n without grooves, and d_c and w are normalized by D and H, respectively. As the d_c and w values are increased, the f_0 value of the TM_{11p} mode is decreased significantly. However, the change of f_0 value for the TE_{01p} mode is negligibly small. Thus, we can separate the TM_{11p} mode from the TE_{01p} mode by changing the size of the groove.



(a) TE_{011} mode



(b) TM_{111} mode

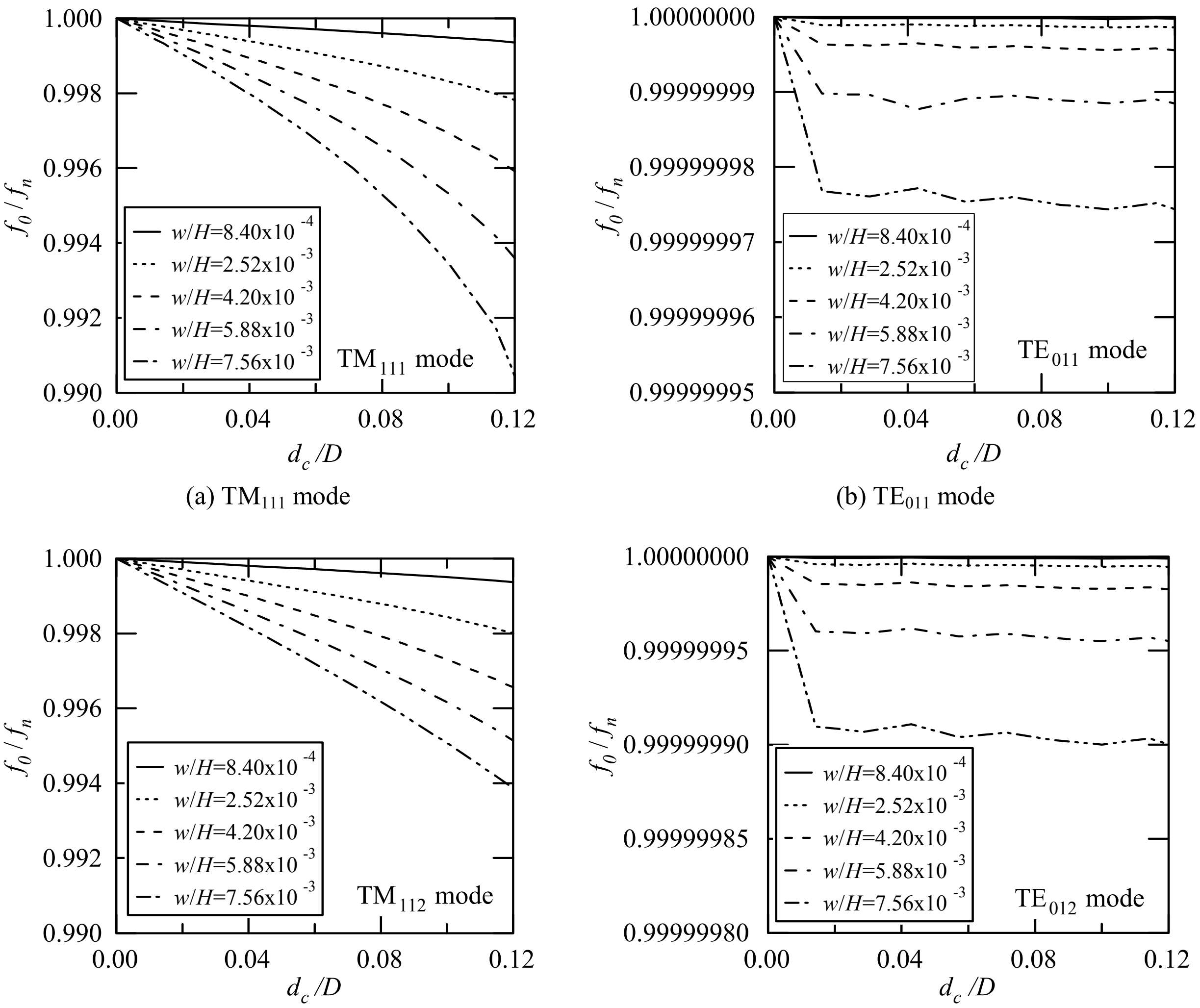
Fig. 3 The convergence of the solution of the TE_{011} and TM_{111} modes with number of expansion modes N (D=7.0mm, H=23.8mm, d_c=0.2mm w=0.2mm)

The ratio of d_c/D and w/H were determined to be 0.085 and 6.3×10^{-3}, respectively, so that the TM_{11p} mode decreases about 0.5%, compared with the case without grooves.

4 Measurement

A circular cavity with grooves is manufactured according to the design described above. The dimensions are D= 6.990mm, H=23.770mm, d_c=0.6mm and w=0.15mm. Here D and H are the average values calculated from the measured resonant frequencies of the many TE_{01p} modes [6].

The experiments are conducted with two cavities. The first cavity is the one with grooves described above. Then



(a) TM_{111} mode (b) TE_{011} mode

(c) TM_{112} mode (d) TE_{012} mode

Fig. 4 Variation of the resonant frequencies of the TE and TM modes with the groove depth d_c and width w

the planer conductor plates at the both ends shown in Fig. 1 are replaced with two convex conductor plates shown in Fig. 5. In this case, a cavity without grooves having D=6.990mm, H=23.470mm is formed.

These cavities are excited and detected by a pair of coaxial cables with small loops at their top ends. The plane of the small loops is rotated by 45 degrees to z-axis to excite both the TE_{01p} and TM_{11p} modes. The measured results are shown in Fig. 6(a) and (b) for the cavities with and without grooves, respectively. The resonant frequencies calculated from the computer program are indicated on the top of Fig. 6. Moreover, the comparison between calculated and measured f_0 values for the TM_{112} mode is shown in Table 1.

In Fig. 6(a), it is seen that the degenerate TM_{112} mode is separated from the TE_{012} mode due to the air-gap effect between the convex conductor plate and the cylinder. On the other hand, it is found from Fig. 6(b) that the f_0 of the TE_{012} mode is shifted a little due to the reduction of H from 23.770mm to 23.470mm. The resonant frequency of the TM_{112} mode is decreased significantly, and is very close to the designed value.

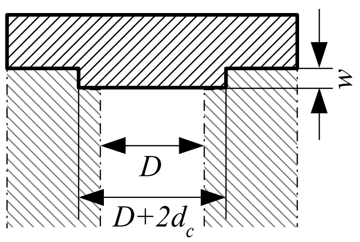
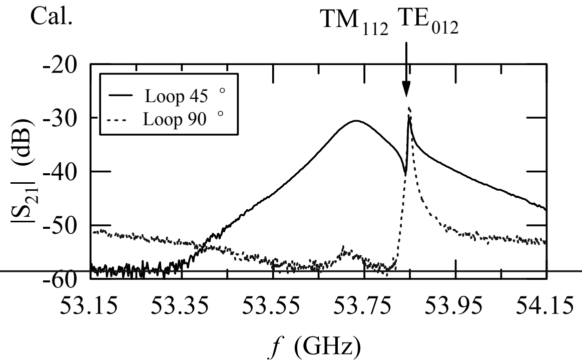
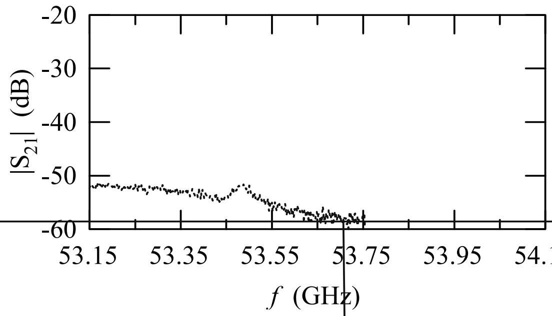


Fig. 5 Cross sectional view of a convex conductor plate

In actual measurement for dimension and relative conductivity, the plane of the small loops is rotated by 90 degree to z-axis to excite only the TE_{01p} modes. The frequency responses are shown in Fig. 6(a) and (b) by dash lines, respectively. The comparison of measured results of cavity without and with grooves for the TE_{012} mode is





shown in Table 2. It is found that the value of Q_u for cavity with grooves is larger than one without grooves, and their values have a small scatter.

Thus, the grooves machined at both ends are effective in separating the degenerate TM_{11p} mode from the TE_{01p} mode.

5 Conclusion

It is verified numerically and experimentally that the grooved circular cavity is useful to separate degenerate TM_{11p} mode from the TE_{01p} mode. The computer program by the mode matching method is powerful in designing such a grooved circular cavity.

Acknowledgements

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Table 1 Comparison between calculated and measured f_0 values for the TM_{112} mode

Cavity Type & Dimension (mm)	Calculated f_0 (GHz)	Measured f_0 (GHz)	Error (%)
No groove D=6.990, H=23.470	53.847	53.733	0.2
Groove d_c=0.6, w=0.15 D=6.990, H=23.770	53.567	53.471	0.2
Shift of f_0 (GHz)	0.280	0.262	6.4

Table 2 Comparison between cavity without grooves and with grooves for the TE_{012} mode

Cavity Type	Calculated f_0 (GHz)	Measured f_0 (GHz)	Measured Q_u
No groove D=6.990, H=23.470	53.847	53.842 ±0.001	10940 ±950
Groove d_c=0.6, w=0.15 D=6.990, H=23.770	53.809	53.816 ±0.001	11450 ±290

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